**Tutorial Activity 6**

**Week 7**

1. Answer the following questions:

(a) What stylised features of financial data cannot be explained using linear time series models?

*-Frequency*: Stock market prices are measured every time there is a trade or somebody posts a new quote, so often the frequency of the data is very high

*-Non-stationarity*: Financial data (asset prices) are covariance non-stationary; but if we assume that we are talking about returns from here on, then we can validly consider them to be stationary.

-*Non-normality*: They are not normally distributed – they are fat-tailed.

-*Volatility pooling and asymmetries in volatility*: The returns exhibit volatility clustering and leverage effects.

Of these, we can allow for the non-stationarity within the linear (ARIMA) framework, and we can use whatever frequency of data we like to form the models, but we cannot hope to capture the other features using a linear model with Gaussian disturbances.

(b) Which of these features could be modelled using a GARCH(1,1) process?

The GARCH models are designed to capture the volatility clustering effects in the returns (GARCH(1,1) can model the dependence in the squared returns, or squared residuals), and they can also capture some of the unconditional leptokurtosis, so that even if the residuals of a linear model of the form given by the first part of the equation in part (d), the s, are leptokurtic, the standardised residuals from the GARCH estimation are likely to be less leptokurtic. Standard GARCH models cannot, however, account for leverage effects.

(c) Why, in recent empirical research, have researchers preferred GARCH(1,1) models to pure ARCH(*p*)?

This is essentially a “which disadvantages of ARCH are overcome by GARCH” question. The disadvantages of ARCH(q) are:

- How do we decide on q?

- The required value of q might be very large

- Non-negativity constraints might be violated.

When we estimate an ARCH model, we require

>0 ∀ (since variance cannot be negative)

The GARCH(1,1) goes some way to get around these. The GARCH(1,1) model has only three parameters in the conditional variance equation, compared to for the ARCH() model, so it is more parsimonious. Since there are less parameters than a typical th order ARCH model, it is less likely that the estimated values of one or more of these 3 parameters would be negative than all parameters. Also, the GARCH(1,1) model can usually still capture all of the significant dependence in the squared returns since it is possible to write the GARCH(1,1) model as an ARCH(∞), so lags of the squared residuals back into the infinite past help to explain the current value of the conditional variance, .

(d) Consider the following GARCH(1,1) model

, mean equation

variance equation

If is a daily stock return series, what range of values are likely for the coefficients , , , and ?

Since are returns, we would expect their mean value (which will be given by ) to be positive and small. We are not told the frequency of the data, but suppose that we had a year of daily returns data, then μ would be the average daily percentage return over the year, which might be, say 0.05 (percent). We would expect the value of again to be small, say 0.0001, or something of that order. The unconditional variance of the disturbances would be given by /(1-( + )). Typical values for and are 0.15 and 0.8 respectively.

The important thing is that all variance parameters must be positive, and the sum of and would be expected to be less than, but close to, unity, with > .**?????????????**

(e) Compare and contrast the following models for volatility, noting their strengths and weaknesses.

1. Historical volatilty
2. GARCH family of models

There are of course a large number of competing methods for measuring and forecasting volatility, and it is worth stating at the outset that no research has suggested that one method is universally superior to all others, so that each method has its merits and may work well in certain circumstances. Historical measures of volatility are just simple average measures - for example, the standard deviation of daily returns over a 3-year period. As such, they are the simplest to calculate, but suffer from a number of shortcomings. First, since the observations are unweighted, historical volatility can be slow to respond to changing market circumstances, and would not take advantage of short-term persistence in volatility that could lead to more accurate short-term forecasts. Second, if there is an extreme event (e.g. a market crash), this will lead the measured volatility to be high for a number of observations equal to the measurement sample length. For example, suppose that volatility is being measured using a 1-year (250-day) sample of returns, which is being rolled forward one observation at a time to produce a series of 1-step ahead volatility forecasts. If a market crash occurs on day t, this will increase the measured level of volatility by the same amount right until day t+250 (i.e. it will not decay away) and then it will disappear completely from the sample so that measured volatility will fall abruptly.

GARCH models overcome this problem with the forecasts as well, since a GARCH model that is “stationary in variance” will have forecasts that converge upon the long-term average as the horizon increases (see part (a) of this question). GARCH models will also overcome the two problems with unweighted averages described above. However, GARCH models are far more difficult to estimate than the other two models, and sometimes, when estimation goes wrong, the resulting parameter estimates can be nonsensical, leading to nonsensical forecasts as well. Thus it is important to apply a “reality check” to estimated GARCH models to ensure that the coefficient estimates are intuitively plausible.

(f) Estimate following models using S&P500 stock market index:

1. Historical volatilty
2. ARCH(1)
3. GARCH(1,1)

See R script file.

(g) Suppose now that the researcher had estimated the GARCH(1,1) model for a series of returns on a stock index and obtained the following parameter estimates: = 0.0023, , , . Inspect the persistence and postivity condition.